On Signal to Noise Ratio Tradeoffs in fMRI

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This monograph addresses the question of signal to noise ratio (SNR) in fMRI scanning, when parameters are changed under conditions of constant total scan time. While the results are presented in terms of spiral methods, the analysis itself is general.

I. Introduction

When planning an fMRI experiment, it is desirable to optimize the SNR to maximize the statistical return on scan time investment. At first glance, the tradeoffs may seem unclear. As more slices are acquired, the TR needed to fit in the slices increases linearly. With longer TR the flip angle and degree of magnetization recovery between pulses both increase, leading to higher signal. However, at constant scan time, fewer time frames can be acquired, leading to lower statistics in the post-processing analysis. It is not obvious which of these effects dominates: increasing signal with longer TR, or decreased overall SNR with fewer frames.

Similarly, the spiral sequence allows interleaving to reduce the readout time, which can be beneficial in reducing blurring and distortion. However, the use of interleaves also reduces the number of time frames that can be acquired. Is there a SNR penalty?

II. SNR in fMRI

The SNR in a single image acquired with an interleaved method is given by

$$SNR_o = C' \,\Delta x\, \Delta y\, \Delta z \sqrt{n_I Tad}\;\; f(TR,TE), \qquad (1)$$

where C' is a constant, Δx, Δy, Δz are the dimensions of the imaging voxel, n_I is the number of interleaves (which might be 1 for either EPI or spiral), Tad is the duration of the acquisition readout ("A/D time"), and f is a function that describes the NMR signal dependence on the relaxation characteristics of the tissue, given for gradient-recalled echo method by

$$f(TR,TE) = \frac{(1-E)\,\sin(\alpha)}{1-\cos(\alpha)\,E}\, e^{-TE/T2^*}, \qquad (2)$$

where TR and TE are scanner parameters with the usual connotation, T2* is the transverse relaxation time, α is the flip angle, and E is given by

$$E = e^{-TR/T1}. \qquad (3)$$

It is known that the signal is maximized when α is chosen as the Ernst angle, $\cos(\alpha_e) = E$. This can be shown by differentiating Eq. (2) with respect to α and setting the result equal to zero. With this choice, Eq. (2) can be written

$$f(TR,TE) = \sqrt{\frac{1-E}{1+E}}\ e^{-TE/T2^*}. \quad (4)$$

Thus, Eqs. (1) and (4) describe the SNR in a MR image.

However, the real interest is in the BOLD signal, which is related to the sensitivity of the image intensity to small changes in the relaxivity of the tissue, R2* = 1/T2*, caused by BOLD modulation. Accordingly, the BOLD signal S is given by

$$S = dI/dR2^*\ \Delta R2\ , \quad (5)$$

where I is the signal intensity, proportional to the SNR given by (1) and (4), and $\Delta R2^*$ is the change in relaxivity that causes the BOLD contrast changes. It can be shown that S is maximized when the echo time TE is set equal to T2* by differentiating Eq. (5) with respect to TE and setting the result equal to zero. With this choice of echo time, we have that the BOLD SNR is given by

$$BSNR_o = C''\ \Delta x\ \Delta y\ \Delta z \sqrt{n_{\ell} Tad\ \frac{1-E}{1+E}}\ , \quad (6)$$

where C" is another constant. To recap, Eq. (6) is valid and the signal is maximized when the flip angle is chosen as α_e and the echo time is chosen as T2*.

Now for interleaved spiral acquisitions, the equivalent resolution (matrix size) in the slew-rate limited case is given by (1)

$$N = \left\{\left(\frac{3\ N_{\ell}\ Tad}{2}\right)^2 \gamma\ D So\right\}^{1/3}, \quad (7)$$

where γ is the gyromagnetic ratio for protons, D is the FOV, and So is the gradient slew rate. Thus, for a given resolution N_{ℓ} Tad is a constant. Then, (6) becomes

$$BSNR_o = C\ \Delta x\ \Delta y\ \Delta z \sqrt{\frac{1-E}{1+E}}\ . \quad (8)$$

III. Total SNR

In an fMRI experiment, N_f frames are acquired. While there are numerous statistical analysis methods that can be utilized to generate the activation map, most have the common feature that the final SNR in the post-processed activation map can be calculated as if the image frames are added incoherently. This is approximately true even if there is significant temporal autocorrelation between contiguous time samples (due to the slow hemodynamic filter function), since the background noise in unactivated pixels is suppressed incoherently and thereby reduces the type II errors (false positives). Thus the total SNR (or Z score) will increase approximately as the square root of N_f, assuming the background noise is indeed uncorrelated from frame to frame. (This assumption may fail at higher field strengths where noise from basal metabolic processes in the gray matter may dominate the RF "Johnson" noise; we will not consider this issue here.)

Let T_S be the total scan time. Then, the number of frames is

$$N_f = T_s/n_I\, TR, \qquad (9)$$

and the total BOLD SNR for the scan is given by (denoted simply as SNR)

$$SNR = BSNR_o \sqrt{N_f} = C\, \Delta x\, \Delta y\, \Delta z \sqrt{\frac{T_s}{N_I\, TR}\, \frac{1-E}{1+E}}\ . \qquad (10)$$

Note that the SNR decreases as the number of interleaves increases, and increases as the square root of the total observation time, T_S.

Equation (10) allows us to determine the answer to the question originally posed: How does the SNR vary as the TR is changed? The answer is shown in Fig. 1.

IV. Experiment

Scans were performed using a single-shot spiral sequence with resolution of 96x96, 24 cm FOV, 5 mm slices, TE = 30 ms, Bo = 3T, with the subject performing bilateral finger tapping with a 20s on/ 20s off block paradigm, for 200 s. Four scans were obtained using the following values of TR/FA/N_f:

TR	N_f	FA, deg
3000	67	83
2000	100	75
1000	200	60
500	400	45

The flip angle was chosen as approximately the Ernst angle, with T1 = 1400 assumed for gray matter at 3T. The data were analyzed using cross correlation with a sine wave and displayed as z-scores.

Results for 4 of 6 slices are shown in Fig. 2 as raw z-score maps. The average z-scores in activated pixels were 1.97, 2.32, 2.80, 2.93, respectively.

V. Discussion

As may be seen in the Fig. 1, the SNR is optimized by making TR as short as possible, although the dependence is rather modest when TR is ≤ 2 T1. What this means in practice is that only as many slices as are necessary should be chosen, because the larger the number of slices, the longer TR, and the smaller the SNR. This also implies that a 3D acquisition, which has the shortest TR (the minimum TR is independent of the number of slices and is the same as a 1 slice 2D acquisition), will have superior SNR to a 2D acquisition when compared at some number of slices. Unfortunately, at this time the inherent SNR advantage of 3D techniques may not be realized, because other noise effects having to do with motion, may sometimes dominate.

A second consequence of (10) is that increasing the number of interleaves decreases the SNR. That is, because N_I Tad is a constant, Tad decreases, and (7) shows that the SNR per image is constant if the resolution is fixed. With more interleaves, there are fewer time frames and the total SNR decreases. Thus, increasing the number of interleaves should only be done if it is needed to avoid T2* dropouts or blurring from off-resonance.

A third consequence is that longer T1 decreases the SNR. This unfortunately occurs at higher field strength. This may be seen by evaluating (10) in the limit of TR = 0:

$$SNR = C\,\Delta x\,\Delta y\,\Delta z\,\sqrt{\frac{T_s}{2\,N_I\,T1}}\;.$$

Thus, some of the SNR gain from higher field strength is offset by this effect.

With very short TR, there may be disadvantages from in-flow effects and additional motion artifacts in end slices from saturation of the imaging volume.

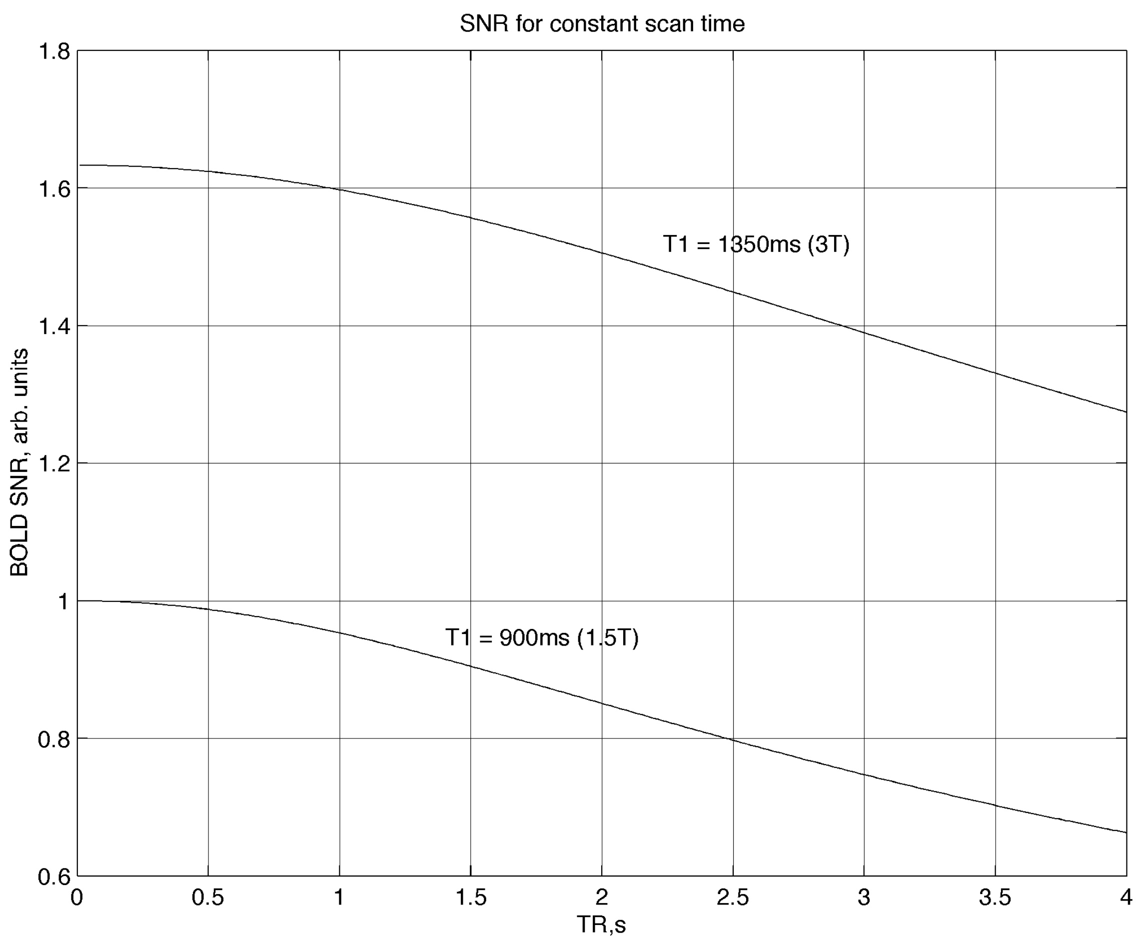

FIGURE 1. Plot of SNR for GRE acquisition under conditions of constant scan time. The SNR is monotone decreasing for increasing TR, which implies that the number of slices (and consequently TR) should be kept as small as is necessary to cover the desired region. This calculation assumes the SNR increases 2x for 3T and uses T1 values appropriate to the two field strengths.

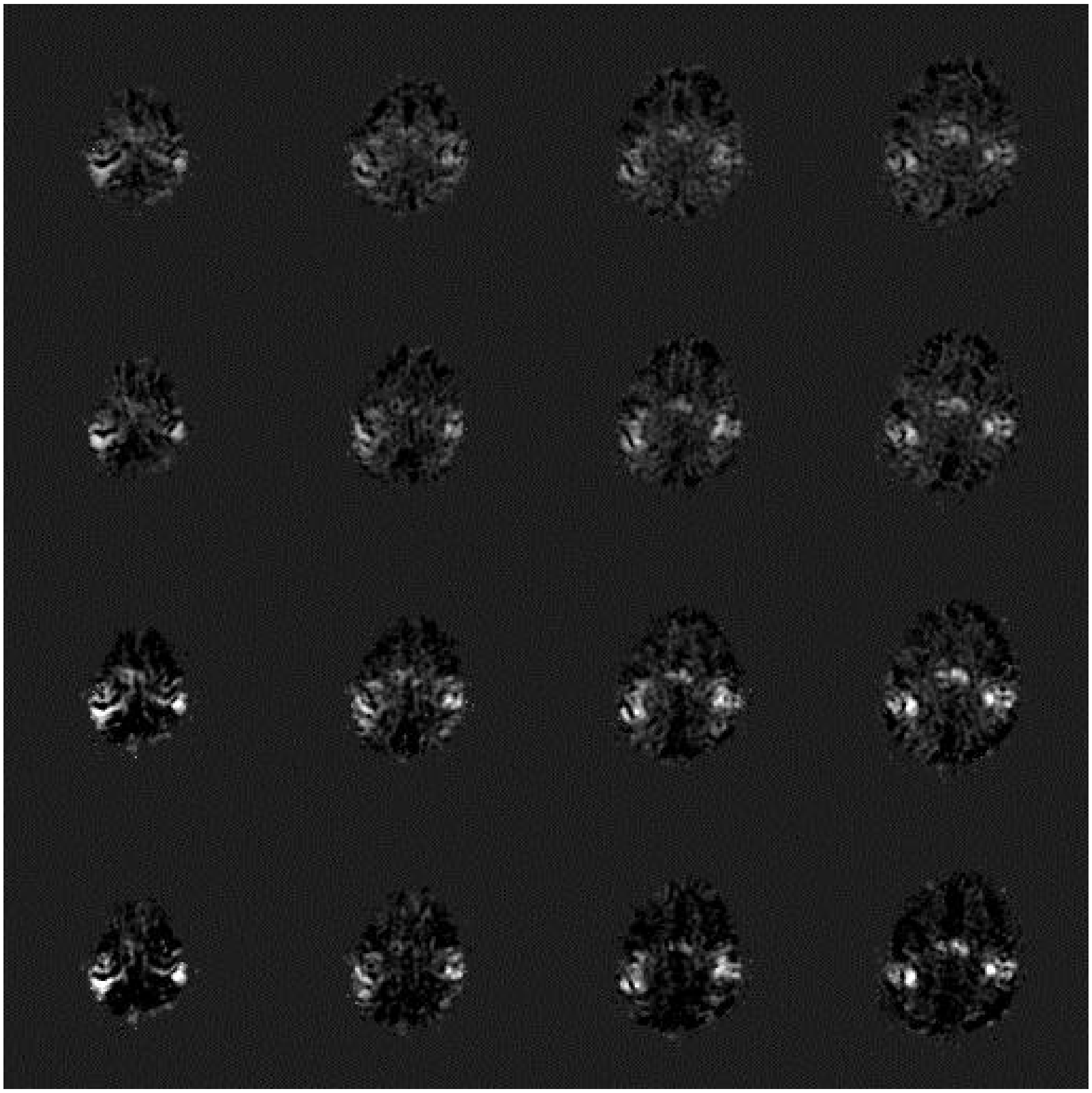

FIGURE 2. Activation maps for 4 slices (columns) for TR = 3s, 2s, 1s, 0.5s (top row to bottom row, respectively). The advantage of shorter TR is manifest as higher z-scores (bottom rows). The effect of decreasing TR from 1s to 0.5 s is small, as expected from Fig. 1.

Reference

1. Glover, G.H. Simple analytic spiral k-space algorithm. Magn. Reson. Med.42:412-415 (1999).